CREMONA
CONFERENCE
BASEL 2016

Question session held on Thursday, 16.09.2016

List of the 10 question asked

1) Find classification of finite subgroups G of Cremona group $\mathrm{Bir}(\mathbb{P}^2)$ up to conjugation that are incompressible (every dominant G-equivariant rational map $\mathbb{P}^2 \dashrightarrow Z$ with a faithful rational action of G on Z is birational). Do such groups exist?

Already asked by Vladimir Popov in [Pop2014, Question 9, page 211].

2) More generally, can we describe the graph associated to conjugacy classes of subgroups of a Cremona group isomorphic to a given finite group? An arrow corresponds to a G-equivariant dominant rational map.

Asked by Alexander Duncan

3) Let $\mathcal{L}$ be a complete linear system of surfaces of degree d in $\mathbb{P}^3$ of dimension $r > 0$ (complete = complete on the blow-up) whose general member S is irreducible with desingularization of geometric genus $p_g = 0$. Is there a bound from above for r?

Castelnuovo claims that if $r > 19$, then, up to Cremona transformations, $\mathcal{L}$ has a base line of multiplicity $d-2$ or a base point of multiplicity $d-1$, and if neither of these happen and $r = 19$, then, up to Cremona transformations, $\mathcal{L}$ is the linear system of cubic hypersurfaces.

Can we figure out a systematic way of decreasing the degree of a surface by Cremona transformations? Or find examples of (linear systems of) surfaces whose degree cannot be decreased by Cremona transformations... (See examples of this type in [MelPol2012])

Asked by Ciro Ciliberto

4) Is $\mathrm{Bir}(\mathbb{P}^3)$ simple (as an abstract group)? Same question for $\mathrm{Bir}(\mathbb{P}^n)$, $n \geq 4$.

Already asked by Federigo Enriques and Vasily A. Iskovskikh in [Enr1895, page 116] *and* [Isk1987].

5) For $n \geq 3$, is $\mathrm{Bir}(\mathbb{P}^n)$ generated by the linear and the de Jonquières maps? (True for $n = 2$, probably false for higher dimension.)

Already asked by Ivan Pan and Aron Simis in [PanSim15] .

Is it true if one add transformations preserving families of congruences of order 1 and automorphisms of Fano varieties?

6) For any $n \geq 2$ does there exists $r \in \mathbb{N}$ such that all p-subgroups of $\mathrm{Bir}(\mathbb{P}^n)$ with $p \geq r$ are abelian and generated by at most n elements? (Ok for $n = 2$ and $n = 3$.)

Asked by Yuri Prokhorov

7) Find all (rational) Fano 3-folds X of Picard rank 1 containing an open subset isomorphic to $Z \times \mathbb{A}^1$ (such an open subset is called cylinder). (There exist non-negative results)

Asked by Yuri Prokhorov

8) Is the following conjecture true? "Let X_3 be a smooth cubic 4-fold. Then X_3 does not contain cylinders."

Asked by Ivan Cheltsov

9) Let S be a smooth del Pezzo variety of degree 1 or 2. For which ample divisiors H on S do there exist H-polar cylinders?

Asked by Ivan Cheltsov

10) Are there affine varieties X such that $\mathrm{Aut}(X)$ is *not* Jordan?

(By [SheZha2018, Theorem 1.6], for every projective variety Y, the group $\mathrm{Aut}(Y)$ is Jordan. By [Zar2014, Corollary 1.3] there are varieties Z such that the group $\mathrm{Bir}(Z)$ is not Jordan.)

Already asked by Vladimir Popov in [Pop2014, Question 1, page 194].

References

[Enr1895] Federigo Enriques. *Conferenze di Geometria: fundamenti di una geometria iperspaziale.* Bologna, 1895.

[Isk1987] Vasily A. Iskovskikh. *Cremona group.* Encyclopedia of Mathematics. 1987.

[MelPol2012] Massimiliano Mella and Elena Polastri, *Equivalent birational embeddings II: divisors.* Math. Z. **270** (2012), no. 3-4, 1141–1161.

[SheZha2018] Sheng Meng, De-Qi Zhang, *Jordan property for non-linear algebraic groups and projective varieties*, Amer. J. Math. **140** (2018), No. 4, 1133–1145.

[PanSim15] Ivan Pan and Aron Simis. *Cremona maps of de Jonquières type.* Canad. J. Math., **67**(4):923–941, 2015.

[Pop2014] Vladimir L. Popov, *Jordan groups and automorphism groups of algebraic varieties.* Automorphisms in birational and affine geometry, 185–213, Springer Proc. Math. Stat., 79, Springer, Cham, 2014.

[Zar2014] Yuri. G. Zarhin, *Theta groups and products of abelian and rational varieties*, Proc. Edinb. Math. Soc (2) **57**(1) (2014), 299–304.