Conference

On affine algebraic geometry in honour of Hanspeter Kraft

June 2-6, 2014

Karin BAUR (Graz) Thursday 5, 14h00–15h00

Dimer models and categories with Grassmannian structure

A dimer model can be defined as a quiver with an embedding of it into a compact oriented surface, giving rise to a tiling of the surface. We consider such dimer models in surfaces with boundary, in particular, we associate them to alternating strand diagrams (Postnikov). These diagrams give rise to dimer models with boundary, when taking their quivers and relations coming from an associated potential.

We show that the category associated to the Grassmannian by Jensen-King-Su arises from an algebra associated to an alternating strand diagram. This is joint work with A. King (Bath) and R. Marsh (Leeds).

Michel BRION (Grenoble) Thursday 5, 15h15–16h15

On the local structure of algebraic group actions

The talk will present some classical and recent results on local properties of algebraic group actions. For example, every normal variety X equipped with an action of a split torus T is the union of T-stable affine open subsets; if X is non-normal, this still holds for after a finite etale cover.

Corrado DE CONCINI (Roma) Thursday 5, 11h00–12h00

Covariants in the exterior algebra of a simple Lie algebra

For a simple complex Lie algebra $\mathfrak{g}$ we study the space of invariants $A = (\bigwedge \mathfrak{g}^* \otimes \mathfrak{g}^*)^{\mathfrak{g}}$, (which describes the isotypic component of type $\mathfrak{g}$ in $\bigwedge \mathfrak{g}^*$) as a module over the algebra of invariants $(\bigwedge \mathfrak{g}^*)^{\mathfrak{g}}$. As main result we prove that A is a free module, of rank twice the rank of $\mathfrak{g}$, over the exterior algebra generated by all primitive invariants in $(\bigwedge \mathfrak{g}^*)^{\mathfrak{g}}$, with the exception of the one of highest degree. Joint with P. Papi and C. Procesi.

Jan DRAISMA (Eindhoven) Tuesday 3, 16h45–17h45

The Euclidean distance degree of an algebraic variety

(joint work with Emil Horobet, Giorgio Ottaviani, Bernd Sturmfels, and Rekha Thomas)

The nearest point map of a real algebraic variety with respect to Euclidean distance is an algebraic function. For instance, for varieties of low rank matrices, the Eckart-Young Theorem states that this map is given by the singular value decomposition. I will discuss a theory of such nearest point maps from the perspective of computational algebraic geometry. The Euclidean distance degree of a variety is the number of critical points of the squared distance to a generic point outside the variety. Focusing on varieties seen in applications, I will present numerous tools for computing this degree, as well as numerous values for concrete varieties.

Hans-Christoph IM HOF (Basel) Monday 2, 17h00–17h45

Hanspeter and Basel

Personal recollections of Hanspeter Kraft and some facts on Mathematics at the University of Basel.

Friedrich KNOP (Erlangen) Wednesday 4, 9h30–10h30

On k-spherical varieties

We report on spherical varieties for arbitrary ground fields k. For char $k = 0$ there is a version of the local structure theorem. This can be used to study equivariant embeddings. (Joint work with Bernhard Krötz und Henrik Schlichtkrull).

Jochen KUTTLER (Alberta)

Modules of differentials for Lie algebras — Tuesday 3, 9h30–10h30

In this talk, I will attempt to introduce/discuss modules of differentials for Lie algebras modelled after the corresponding notion for rings. There even might be some motivation for doing so. This is joint work with Arturo Pianzola.

Frank KUTZSCHEBAUCH (Bern) — Tuesday 3, 11h00–12h00

Holomorphic Linearization

The holomorphic linearization problem asked whether every reductive subgroup of the group of holomorphic automorphisms of complex affine n-space is conjugated to a subgroup of the general linear group, equivalently: Is every holomorphic action of a reductive group on $\mathbb{C}^n$ linear after a suitable change of coordinates?

We will give an overview over the most important developments around this problem. In particular we explain the negative answer by Harm Derksen and the speaker found during their time as assistants of Hanspeter Kraft in Basel. Another highlight is the impact of the Homotopy principle in Complex Analysis (also called the Oka-Grauert-Gromov principle) on Linearization, which we describe beginning from joint work of the speaker and Peter Heinzner all the way to an ongoing joint project of the speaker with Finnur Lárusson and Gerald Schwarz.

Peter LITTELMANN (Köln) — Tuesday 3, 14h00–15h00

Favourable modules, Newton-Okounkov bodies and flat degenerations

We introduce the notion of a favourable module for a complex unipotent algebraic group, whose properties are governed by the combinatorics of an associated polytope. We describe two filtrations of the module, one given by the total degree on the PBW basis of the corresponding Lie algebra, the other by fixing a homogeneous monomial order on the PBW basis. In the favourable case a basis of the module is parameterized by the lattice points of a normal polytope and the polytope itself can be recovered as a Newton-Okounkov body of the projective closure of an orbit of the unipotent group.

Roland LOETSCHER (München) — Wednesday 4, 11h00–12h00

Essential dimension of separable subalgebras

Let G denote a linear algebraic group defined over a field F and X a quasi-projective G-variety. For any G-torsor T over a field extension K/F we can form the twist of X by T by taking the geometric quotient $(X \times T)/G$. The G-variety X is called versal, if rational points are dense in every twist of X by a G-torsor (defined over an infinite field K). The essential dimension of G, denoted ed(G) is the least value of $\dim(X) - \dim(G)$, where X ranges over all versal generically free G-varieties. It is a measure of the complexity of the torsors of G.

In this talk we fix a central simple algebra A and a separable subalgebra B and consider the normalizer $G := N_{GL_1(A)}(GL_1(B))$. Its torsors over a field extension K/F correspond to separable subalgebras B' of A_K of the same type of B. We compute ed(G) under some restrictions on the type of B and the degree and index of A. This problem is closely related to computing the essential dimension of projective linear groups and symmetric groups, which have been two of the main sources of motivation for studying essential dimension since its introduction by J. Buhler and Z. Reichstein in 1995.

Lucy MOSER-JAUSLIN (Dijon) — Monday 2, 14h00–15h00

Some examples of automorphism groups of hypersurfaces of affine complex four-space

In this talk, we will review some results on automorphism groups of affine threefolds obtained by affine modifications from affine threespace. We will be particularly interested in some cases where there exist automorphisms of the hypersurface which do not extend to the ambient space. This happens, in particular, in certain situations where the threefold is realised as a fibration over the affine line with a non connected fiber.

Claudio PROCESI (Roma) Thursday 5, 9h30–10h30

Lie algebras and classical invariant theory

We revisit the classical Amitsur–Levitzki theorem in the context of the study of the exterior algebra $\bigwedge \mathfrak{g}$ of a simple Lie algebra g as representation.

We show that the classical theory of Chevalley transgression extends to produce a remarkable theorem on the structure of the isotypic component of the adjoint representation $\mathfrak{g}$ in $\bigwedge \mathfrak{g}$.

Peter RUSSELL (McGill) Friday 6, 11h00–12h00

Affine n-space: the problems are still challenging

I will discuss some advances, both recent and not so recent, and mainly modest, in the general area of morphisms with affine space fibers.

Gerald SCHWARZ (Brandeis) Thursday 5, 16h45–17h45

Hanspeter Kraft, a mathematical appreciation

We recall some highlights of Hanspeter Kraft's contributions to mathematics.

Nolan WALLACH (San Diego) Tuesday 3, 15h15–16h15

Invariant theory for products of classical groups

We consider a group that is a product of classical groups acting on the outer tensor product if the "identity representation" of each of its factors. Using an outer tensor product of Schur-Weyl duality we study polynomial invariants for this representation. In the case when all of the groups are the same we show that there are beautiful formulas for the dimensions of the spaces of invariants of fixed degree as a function of the number of factors. The case when the groups are products of special linear groups is of particular interest to physicists (so-called qudits) was studied with Gilad Gour.

David WRIGHT (St. Louis) Friday 6, 9h30–10h30

Amalgamations and Automorphism Groups

Many types of automorphism groups in algebra have nice structures arising from actions on combinatoric spaces. We review some examples including Nagao's Theorem and the Jung-Van der Kulk Theorem.

For K a field of characteristic zero, it is shown the the tame subgroup $\mathrm{TA}_3(K)$ of the group $\mathrm{GA}_3(K)$ of polynomials automorphisms of $\mathbb{A}^3_K$ can be realized as a generalized amalgamated product, specifically, the product of three subgroups, amalgamated along pairwise intersections, in a manner that generalizes the well-known amalgamated free product structure of $\mathrm{TA}_2(K)$ (which coincides with $\mathrm{GA}_2(K)$ by Jung's Theorem). The result follows from defining relations for $\mathrm{TA}_3(K)$ given by U. U. Umirbaev.

Mikhail ZAIDENBERG (Grenoble) Monday 2, 15h15–16h15

Flexible varieties and infinitely transitive actions

Given a projective algebraic variety X, the identity com- ponent $\mathrm{Aut}_0(X)$ is known to be an algebraic group. This is not true any longer for an affine algebraic variety. Already for the affine space $\mathbb{A}^3$, the structure of the group $\mathrm{Aut}_0(\mathbb{A}^3)$ remains rather mysterious.

In the talk, we will introduce a class of quasi-affine varieties called *flexible*, and show that the action of $\mathrm{Aut}_0(X)$ on such an X is m-transitive for any m. We will provide some constructions, applications, and concrete examples (Gizatullin surfaces, the affine cones over flag varieties, over certain del Pezzo surfaces, over some Fano varieties, etc.).
